Axial Stiffness of Bars

Definition: It is the force required to attain a unit deformation. It is dependent on geometry and material of structural element and independent on loads.

i.e.; Hook's Law $\Rightarrow P = K.\Delta$; $\Delta = 1 \Rightarrow K = P$.

OR $P = 1 \Rightarrow K = \frac{1}{\Delta}$.

From Hook's Law:- $P = K.\Delta$.

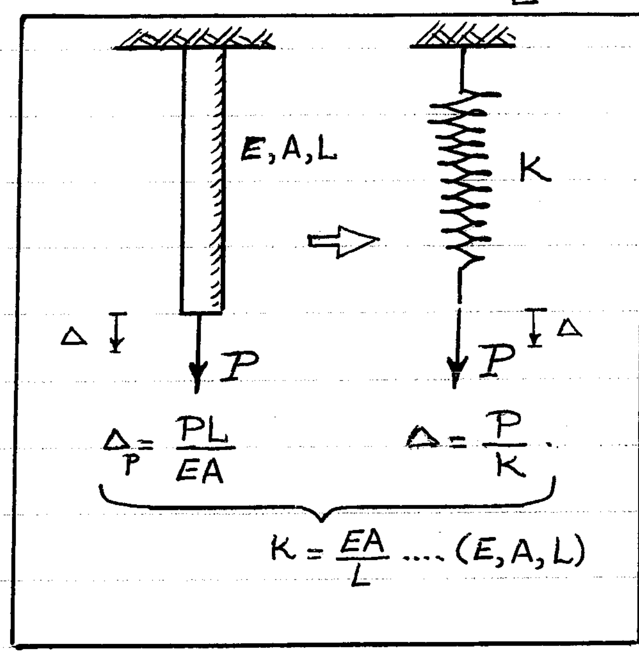
- $P = 1 \Rightarrow K = \frac{1}{\Delta}$.
- $P = P \Rightarrow K = \frac{P}{\Delta_P}$.

But $\Delta_P = \frac{PL}{EA} \Rightarrow K = \frac{P}{\left(\frac{PL}{EA}\right)}$.

$\Rightarrow K = \frac{EA}{L}$.

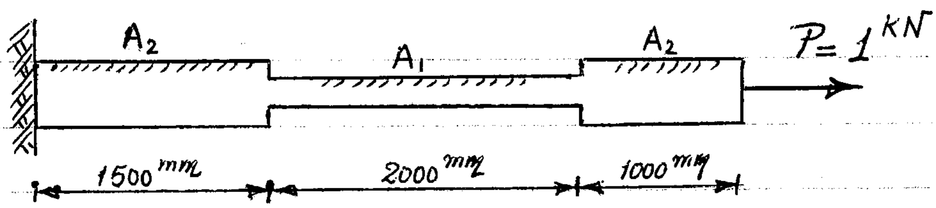
$$\therefore \boxed{K = \frac{EA}{L} = \frac{1}{\Delta}}$$

When bar is subjected to a unit axial load ($P=1$).



Examples.

(Previously Solved).



- Remove all applied loads.
- Apply a unit axial load at tip.

Given: $A_1 = 1000\ mm^2$; $A_2 = 2000\ mm^2$

$E = 200\ GPa = 200\ \frac{kN}{mm^2}$.

- $\Delta = \sum_{i=1}^{3} \frac{PL}{EA} = \frac{1(1500)}{200 * 2000} + \frac{1(2000)}{200 * 1000} + \frac{1(1000)}{200 * 2000} = 0.01625\ mm$

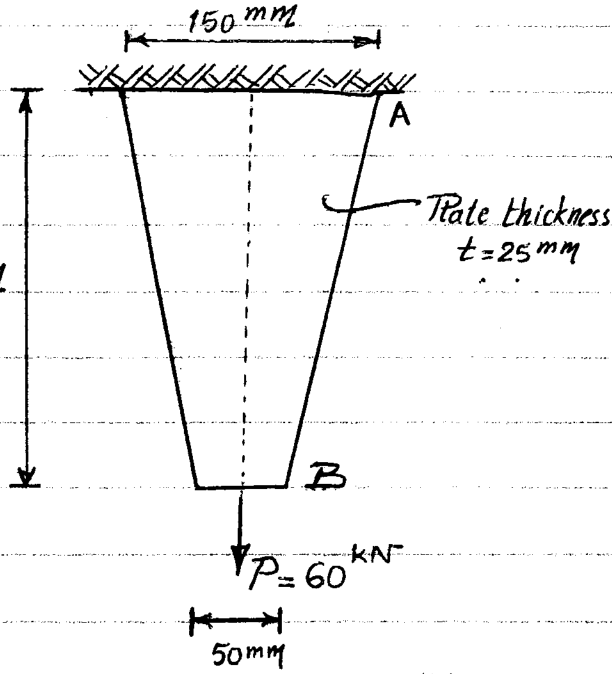
- $K = \frac{1^{kN}}{\Delta} = \frac{1^{kN}}{0.01625\,mm} = \underline{\underline{61.54\ kN/mm}}$.

* Previously solved exampled.

[Tapered Rod].

we have computed Δ_p for $P = 60^{kN}$ acting at end B of the bar. $L = 3^m$

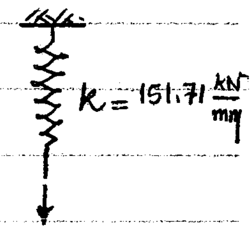
($\Delta_P = 0.3955$ mm).



$$K = \frac{P}{\Delta_P} = \frac{60^{kN}}{0.3955^{mm}}$$

$$\Rightarrow \underline{\underline{K = 151.71 \ \frac{kN}{mm}}}$$

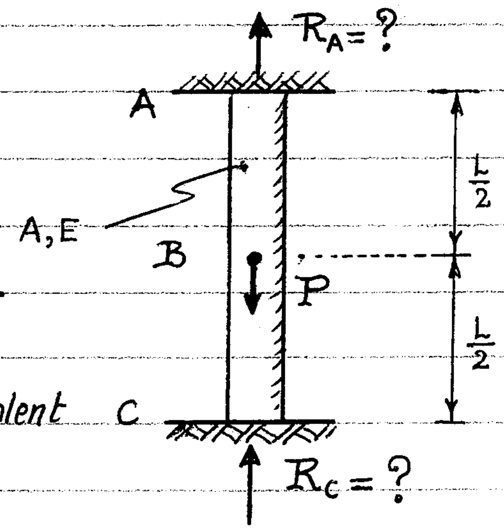
∴ This tapered bar is equivalent (interms of axial stiffness) to a linear elastic spring of spring constant K equals to 151.71 kN/mm.



Analysis Of Statically Indeterminate Bars

There are many problems in which the internal forces can not be determined from statics alone; i.e., drawing a free-body diagram of the member and writing the corresponding equilibrium equations. The equilibrium equations must be complemented by relations involving <u>deformations</u> obtained by considering the geometry of the problem. Because statics is not sufficient to determine either the external reactions or the internal forces, such <u>indeterminate problems</u> can not be solved without the aid of satisfying "complementary conditions" related to <u>compatible or consistent deformations</u>.

Example

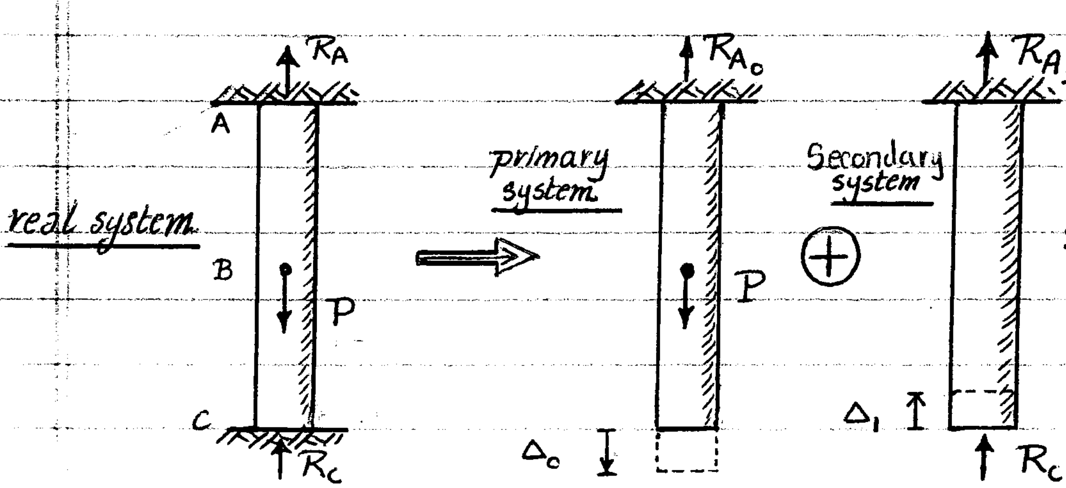


$\Sigma X = 0 \Rightarrow R_A + R_c = P.$

One equation and two unknowns.

$\Rightarrow$ Statics not sufficient.

The indeterminate system is equivalent to superposition of two statically determinate systems, as shown :-



$R_A = R_{A_0} + R_{A_1}$.

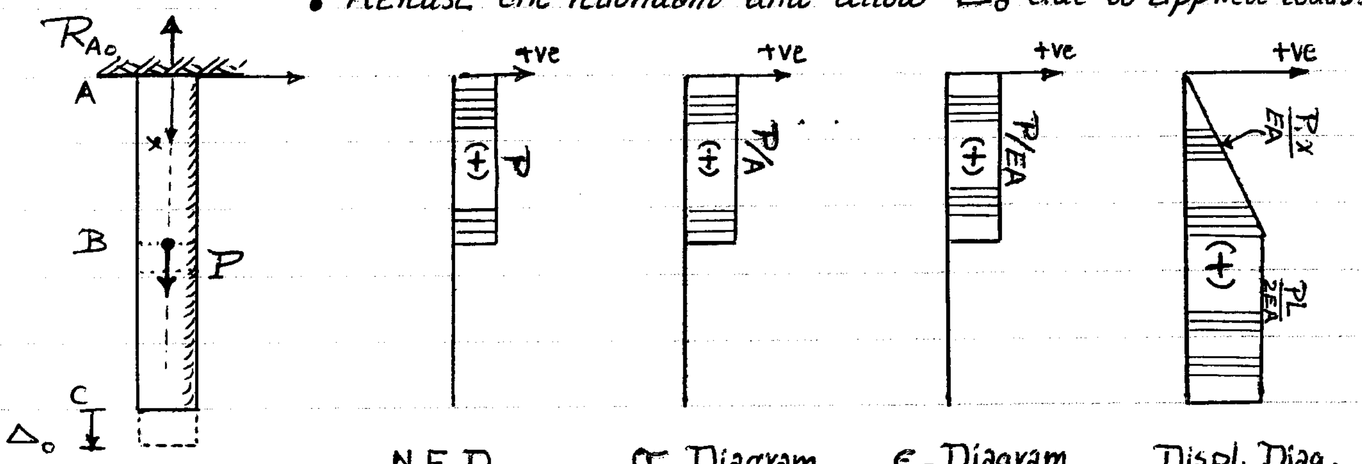
<u>Compatibility Equation:</u>

$\Delta_c = \Delta_0 + \Delta_1 = 0$

(Fixation at C)

* **Primary System** $\Longrightarrow$ Get $\Delta_o = \checkmark$

• Release one redundant and allow Δ_o due to applied loads.



N.F.D (Normal Force Diagram) — σ - Diagram — ϵ - Diagram — Displ. Diag. Relative to A

$\Rightarrow \Delta_0 = \frac{PL}{2EA} \downarrow$ ... (From the Displacement Diagram).

$$\Delta_o = \Delta_A^C = U_c - \cancel{U_A}^{\,0} = \int_0^L \epsilon_x \, dx .$$

OR simply $\Rightarrow \Delta_o = \Delta_A^C = U_c - U_A = \int_0^L \epsilon_x \, dx = \int_0^L \frac{\sigma_x}{E} dx = \int_0^L \frac{P_x}{EA} dx .$

$$\Rightarrow \Delta_o = \sum_i \frac{P_i L_i}{E_i A_i} \quad \ldots \ (2 \text{ segments} \Rightarrow i = 2).$$

$$\Rightarrow \Delta_o = \frac{P_1 L_1}{EA} + \frac{P_2 L_2}{EA} .$$

$L_1 = L_2 = \frac{L}{2}$ & $P_1 = P$ & $P_2 = 0$ $\Rightarrow \Delta_o = \frac{PL}{2EA} \downarrow$ ... (downwards).

* **Secondary System** $\Longrightarrow$ Get $\Delta_1 = \checkmark$

• For the same primary system determine Δ_1 due to R_c.

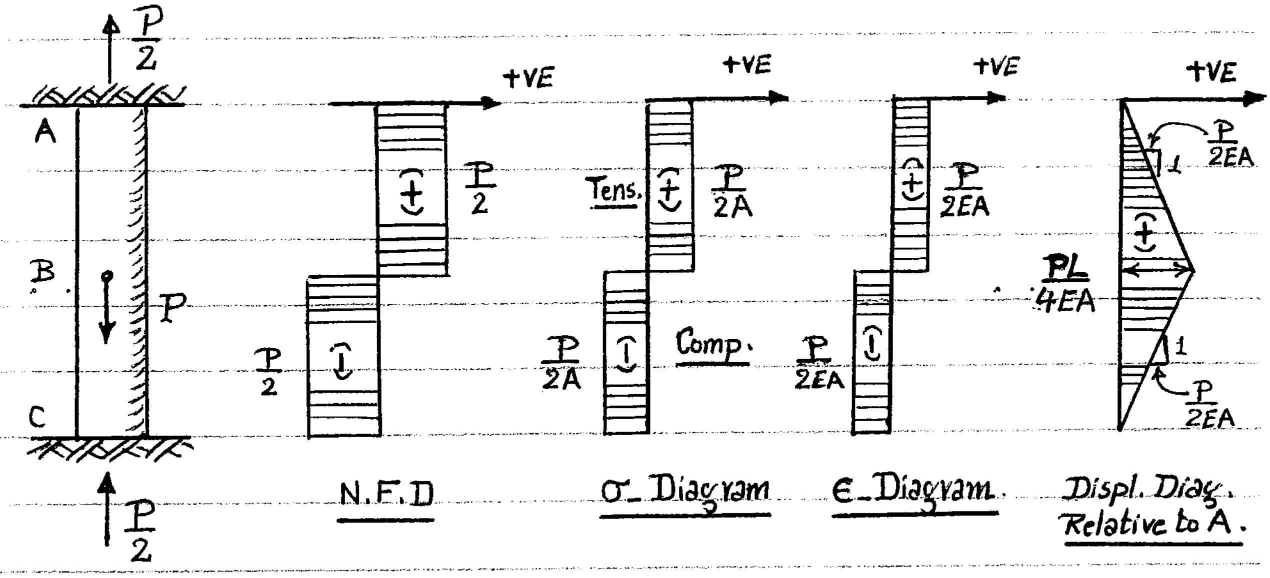
$\Delta_1 = \frac{R_c * L}{EA} \uparrow$ ... (upwards). ..... (One segment).

compatibility equation; consistent deformation at C.

$$\Rightarrow \Delta_c = 0 \Rightarrow \Delta_o + \Delta_1 = 0 \Rightarrow \frac{P\cancel{L}}{2\cancel{EA}} - \frac{R_c \cancel{L}}{\cancel{EA}} = 0 .$$

$\Rightarrow R_c = \frac{P}{2}$ ... (+ve indicates in the assumed sense).

& $R_A = P - R_c = \frac{P}{2}$ OR $R_A = R_{A_o} + R_{A_1} = P + (-R_c) = \frac{P}{2}$.



STEPS

(1) After determining R_A & R_C, Plot N.F.D.

(2) Plot σ_ Diagram ; $\sigma_x = \dfrac{P_x}{A}$.

(3) Plot ε_ Diagram ; $\epsilon_x = \dfrac{\sigma_x}{E} = \dfrac{P_x}{EA}$.

(4) Plot Displacement Diagram Relative to point A.

$$\Delta_A^x = \int_A^x \epsilon_x \, dx = U_x - \cancel{U_A}^{\,0} = \text{Area of strain diagram between A and } x.$$

$$U_B = \frac{P}{2EA} * \frac{L}{2} = +\frac{PL}{4EA}$$

$$U_C = +\frac{PL}{4EA} - \frac{PL}{4EA} = 0 \quad \ldots \text{(Fixed support ✓).}$$

* *Note that slope of the displacement diagram at any pt. must always be equal to the strain at corresponding section.*